Correlation Processing Approach for Eyesafe SLR 2000*

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The SLR 2000 design, discussed in the Canberra meeting two years ago, employs a high PRF / low energy-per-pulse 532 nm transmitter beam which is expanded to fill a large aperture. The system is also designed to be capable of fully unattended operation. The following table summarizes the top level electro-optical parameters for the SLR-2000, as presently configured.

Transmitter Side	**Receiver Side**
Wavelength: 532 nm	Filter Bandpass : 1.2 Å (50% throughput) [B_{opt}]
Energy Per Pulse: 207 μjoule [$E_p\tau_t$]	Quantum Efficiency: 40 % [η]
Pulse Repetition Frequency: 2000 Hz	Total Dark Counts: < 10^4 /second @ 20 °C
Pulsewidth: 140 psec	Resolution: < 100 psec
Transmitter Aperture: 50 cm diam. [D_t]	Receiver Aperture: 50 cm diam. [D_r]
Optical Transmission: > 80 % [τ_t]	Optical Transmission: > 30 % [τ_r]
Half Angle Beamwidth: 20 μrad [θ_t]	Half Angle Beamwidth: 20 μrad [θ_r]
Power Distribution - At Aperture: Top Hat - Far Field: Airy Disc	Field-of-View : Sharp Edged Uniform Sensitivity
Pointing Jitter: < 5 μradians (nominal)	Boresight with Transmitter: < 5 μradians
Pointing Offset: < 5 μradians (nominal)	Special: Quadrant Detector

System eye safety is based on ANSI Z136.1-1993, Section 8.2, Table 5 and the accompanying Errata Sheet. The governing equation is

$$C_{\#}MPE = \frac{E_p\tau_t}{\pi D_t^2 / 4} \qquad (1)$$

for $C_{\#}$ = Multiple pulse correction to the allowable single pulse MPE $= \left[PRF\left(Exposure\,Duration\right)\right]^{-1/4}$, for PRF = laser pulse repetition frequency (Hz),

MPE = Maximum Permissible single pulse/single exposure energy density,

$C_{\#} = \left[PRF\left(0.25\right)\right]^{-1/4}$ (in the green, for short pulses) = 0.2115 for PRF = 2000 Hz,

MPE = 5 (10^{-7}) joules/cm^2 (in the green for short pulses).

Evaluating Equation 1 we find in general that

$$E_p\tau_t = 8.305\left(10^{-8}\right)D_t^2 \quad \text{joules.} \qquad (2)$$

The 50 cm aperture case is used in the baseline SLR-2000 configuration, and corresponds to the 207 μjoule value for energy per pulse at the aperture.

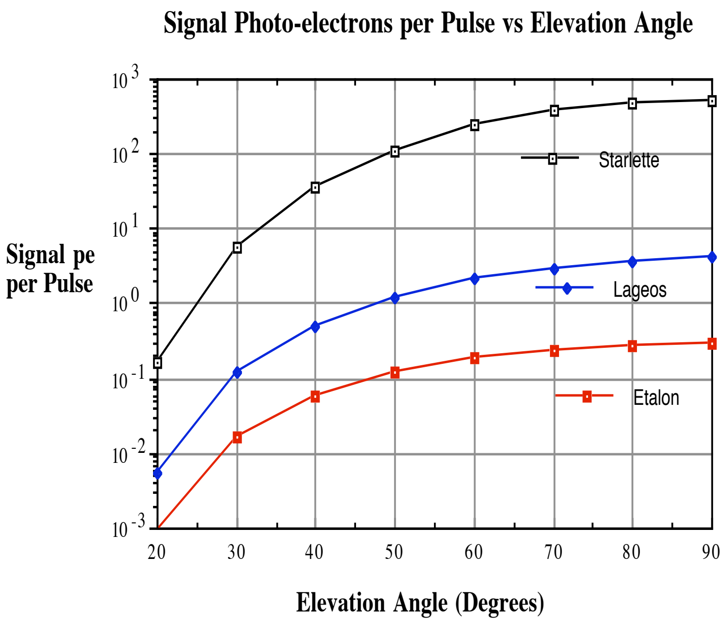
The range equation (cf. Bibliography) is

$$n_{pe}^{s} = \eta\left(\frac{E_p}{h\nu}\right)\eta_t T_a T_c \left[\frac{\sigma}{\pi\theta_t^2\theta_{turb}^2 R^2}\right] T_c T_a \eta_r \left(\frac{A_r}{\pi R^2}\right)\bar{G} \qquad (3)$$

for n_{pe}^{s} = signal photo-electrons (pe's) per pulse; $h\nu$ = energy per photon; T_a = one-way clear atmosphere path transmission; T_{cl} = one-way cirrus cloud path transmission; σ = satellite optical cross-section; θ_{turb} = impact of atmospheric turbulence on beam divergence; R = range from ground station to target; $A_r = \pi D_r^2/4$; $\bar{G}$ = normalized impact of transmitter truncation and beam shape at the satellite = 0.5.

The elevation angle dependence of the parameters and the optical cross sections are shown in the following table for the Starlette, LAGEOS and ETALON satellites. (The cross sections are ~ 80% of typical values, which provides performance margin.) Equation 3 is evaluated for these parameters in the following figure.

Elevation Angle (Degrees)	Range to Starlette(km)	Range to LAGEOS (km)	Range to ETALON (km)	T_a	T_{cl}	θ_{turb}
20	2034	8532	22861	0.75	0.12	1.6
30	1626	7776	21783	0.84	0.36	1.4
40	1362	7164	21006	0.87	0.53	1.27
50	1188	6687	20360	0.89	0.65	1.14
60	1074	6333	19854	0.90	0.72	1.07
70	1002	6089	19491	0.911	0.75	1.03
80	963	5947	19273	0.915	0.77	1.01
90	960	5900	19200	0.92	0.78	1.00
σ (10^6 m^2)	0.52	5.7	48			



During initial acquisition (at the lowest elevation angles), the signal levels per pulse are very small: 0.159 /Starlette /20°, 0.0054 /LAGEOS /20°, and 0.00165 / ETALON / ~ 22°.

There are three functions which the system must perform at these low signal levels: acquisition, ranging, and tracking/pointing during ranging. The assumed requirements during acquisition are shown in the next table --- the basic acquisition requirement is to narrow the initial range, angle and time uncertainties.

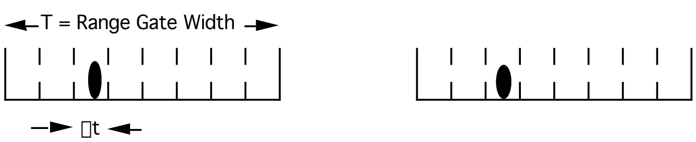
Initial Uncertainties	Starlette	LAGEOS	ETALON	Desired Performance
Angle (μrad)	± 100 x ± 80	± 80 x ± 80	± 80 x ± 80	± 5
Range (nsec)	± 100	± 100	± 100	± 5
Range Rate(nsec/sec)	~ 300	~ 10	~ 1	Target Specific
Time to acquire (sec)	< 10	< 120	< 300	

Ranging must occur with the requirements shown in the next table.

	Starlette	LAGEOS	ETALON
Normal point Precision	~ mm	~ mm	~ mm
Time per Normal Point (sec)	30	120	300

During ranging, slow system pointing errors often occur, and so the final functional requirement is to develop a pointing error from the characteristics of the returned light, and update the pointing angles to enable correcting for these errors.

To meet these requirements at very low signal levels, we use the relative temporal location of the signal pulses over many intervals. The concept is illustrated in the following figure. The basic insight is that signal pulses will reside in time correlated "bins" within the gates, while noise counts will be randomly distributed.



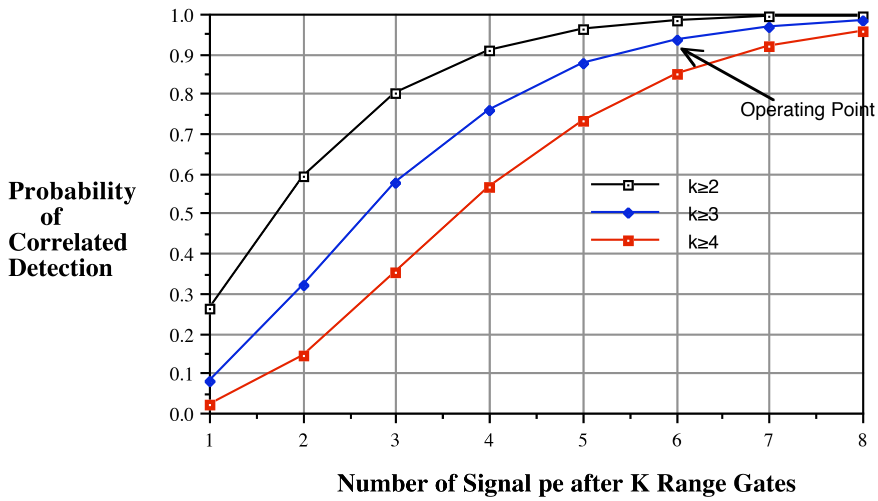
n_{bin} = # of pulse resolution intervals per gate,
N^s_{pe} = mean # of signal pe after K gates
K = # of gates before detection decision; Frame = K range gates
$T = n_{bin}(\Delta t)$ and $N^s_{pe} = K(n^s_{pe})$

We assume that both n^s_{pe} and N^s_{pe} are Poisson distributed. The probability that signal photo-electrons will be detected in corresponding time bins in separate range gates is

$$P_K(\geq 3) = 1 - \left[1 + N^s_{pe} + \frac{\left(N^s_{pe}\right)^2}{2!}\right] e^{-N^s_{pe}}$$, k (# of pe correlations) ≥ 3 . (4)

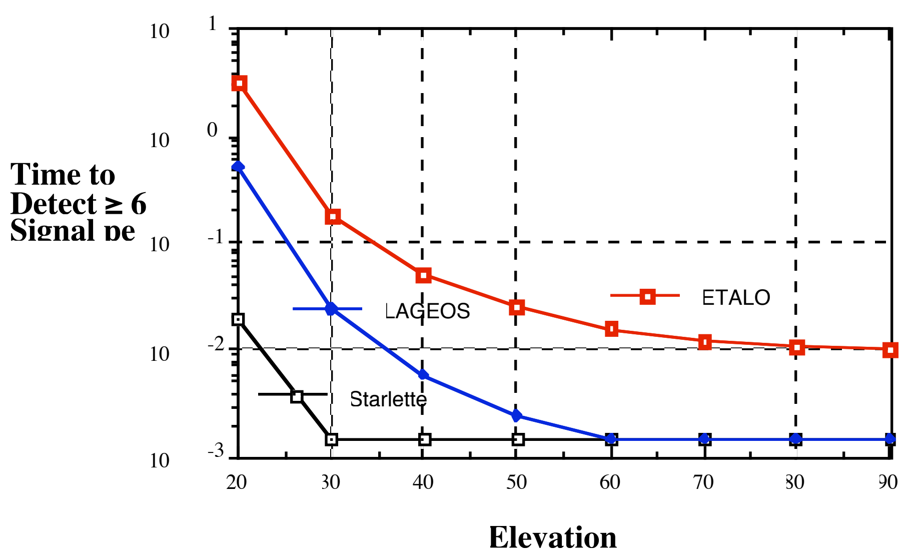
Equation 4, and corresponding ones for k=2 and 4, are evaluated in the following figure.

Correlated Detection Probabilities vs Mean Number of Detected Photo-electrons per Frame



After an average of 6 signal photo-electrons are detected, k ≥ 3 correlation detection provides ~ 94% probability of signal acquisition. The corresponding times required to accumulate the 6 signal photo-electrons are shown in the next figure.

Time to achieve a Mean Photo-electron count ≥ 6 vs Elevation Angle



This figure is interpreted as the time it should take to acquire these satellites, which is a function of elevation angle. The times are: 0.0189 seconds / Starlette / 20°; 0.532 seconds / LAGEOS / 20°; 1.818 seconds / ETALON / 22°. During this same time noise counts occur. Using the formulation in the bibliography, and the parameters in the initial table in this paper, we find for the assumed quadrant photo-detector:

Clear Daytime Optical Background, pe's per range gate:	0.0278 Total

Dark Counts: 10^4 / second , pe's per range gate:		0.00695 per quadrant 0.0020 Total 0.0005 per quadrant
Signal Backscatter:	Prevented by Appropriate Receiver Blanking Programmably vary Laser Firing Time to Prevent "Collisions"	

Net Noise Photo-electron rate:	Day	14.9 / second / quadrant
	Night	1 / second / quadrant

To analyze the probability of false acquisition, we define m = # of noise photo-electrons present in a Frame and n_{bin} = # of time bins per range gate. The probability that these noise photo-electrons will lead to a (false) correlation is given by

$$P_{FAQ} = 1 - \left[e^{-\frac{m}{n_{bin}}} \sum_{j=0}^{k-1} \frac{\left(\frac{m}{n_{bin}}\right)^j}{j!} \right]^{n_{bin}} \qquad (5)$$

m = [noise pe rate] x [time to detect 6 signal pe]

n_{bin} = [(range rate uncertainty) x (time to detect 6 signal pe) $]^{-1}$ x [range gate width].

Equation 5 is evaluated for a single ± 20 μradian beam at minimum elevation angle, k ≥ 3, and a 200 nsec gate in the following table.

		Time for 6 signal pe (sec)	**Range Rate Uncertainty (nsec/sec)**	**m**	**Bin Width (nsec)**	n_{bin}	$P_{FalseAcq}$
Starlette 20°	Day	0.0189	300	0.282	5.67	35	0.025 %
	Night	0.0189	300	0.0192	5.67	35	< 0.00001 %
LAGEOS 20°	Day	0.532	10	7.93	5.32	38	4.75 %
	Night	0.532	10	0.53	5.32	38	0.0017 %
ETALON ~22°	Day	1.818	1	27.1	1.818	110	20.4 %
	Night	1.818	1	1.82	1.818	110	0.0082 %

For Daytime ETALON, the SLR 2000 system parameters and k ≥ 3 correlation detection will find the signal ~ 94% of the time that it is present, and falsely identify noise counts as signal ~ 20% of the time when the signal is absent, for a single ± 20 μradian spot. The total time to acquire includes the effect of scanning the ± 20 μradian beam over the full angular uncertainty, including the effect of spot overlap and revisit, both included in an overhead factor. The resulting times are shown in the next table.

		Initial Angular Uncertainty (μradians)	Time per Spot (seconds)	Approximate Overhead	# of Spots	Maximum Acquisition Time (seconds)	Required Acquisition Time (seconds)
Starlette	Day	± 100 x ± 80	0.0189	33%	27	0.51	< 10
	Night	± 100 x ± 80	0.0189	33%	27	0.51	< 10
LAGEOS	Day	± 80 x ± 80	0.532	50%	24	12.55	< 120
	Night	± 80 x ± 80	0.532	33 %	21	11.17	< 120
ETALON	Day	± 80 x ± 80	1.818	100 %	32	58.2	< 300
	Night	± 80 x ± 80	1.818	33%	21	38.2	< 300

Before ranging, we center the spot on the quadrant to enable tracking, narrow the range gate to 10 nsec, and narrow the time bins to 100 psec . The resulting ranging performance, maintaining k ≥ 3 correlation to correctly identify 100 psec time bins, and defining precision per measurement point: $\boxed{\left(\frac{c\Delta t}{2}\right) \approx 15 \text{ mm}}$, is given in the next table.

Satellite	Elevation Angle (Degrees)	# of Measurement Points per Normal Point	Approximate Precision { 15 mm ÷ $\sqrt{\# \text{ of meas. pts}}$)
Starlette	20	1580	0.38 mm
	90	20,000	0.10 mm
LAGEOS	20	225	1.0 mm
	90	20,000	0.10 mm
ETALON	~ 22	165	1.17 mm
	90	30,000	0.087 mm

Tracking is achieved by using the quadrant signal to provide “slow” pointing updates. Signal counts per quadrant are accumulated until an adequate SNR exists.

$$\mathrm{SNR}_{\mathrm{opt}} = \frac{\mathrm{n}_{\mathrm{pe}}^{\mathrm{s}}}{\left[\mathrm{n}_{\mathrm{pe}}^{\mathrm{s}} + \mathrm{n}_{\mathrm{pe}}^{\mathrm{n}}\right]^{1/2}} \Rightarrow SNR_{opt} = \frac{\dot{n}_{pe}^{s} T_s}{\left[\dot{n}_{pe}^{s} T_s + \dot{n}_{pe}^{n} T_s\right]^{1/2}} = \frac{\dot{n}_{pe}^{s}}{\left[\dot{n}_{pe}^{s} + \dot{n}_{pe}^{n}\right]^{1/2}} T_s^{1/2} \tag{6}$$

for T_S = measurement time. For a 10 nsec range gate, signal spot centered on the quadrant, and a $\mathrm{SNR}_{\mathrm{quadrant}}$ = 10 dB, tracking performance is shown in the next table.

Satellite	**Elevation Angle**	**T_S (seconds)**	**Angular Motion per Update (milli-degrees)**
Starlette (Daytime)	20°	0.127	15
	90°	0.0005	0.22
LAGEOS (Daytime)	20°	4.73	156
	90°	0.005	0.28
ETALON (Daytime)	~ 22°	23.1	230
	90°	0.22	2.53

For realistic system parameters, correlation detection provides :

Satellite Acquisition Probability	≥ 90 %
Initial Acquisition Times: (Daytime)	Starlette ~ 0.5 seconds at 20°
	LAGEOS ~ 13 seconds at 20°
	ETALON ~ 60 seconds at ~ 22°
Ranging Precision per Normal Point: (Daytime)	Starlette ~ 0.4 mm at 20°
	LAGEOS ~ 1 mm at 20°
	ETALON ~ 1.2 mm at ~ 22°
Tracking/Pointing Update Rate: (Daytime)	Starlette ~ every 15 milli-deg at 20°
	LAGEOS ~ every 156 milli-deg at 20°
	ETALON ~ every 230 milli-deg at ~ 22°

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